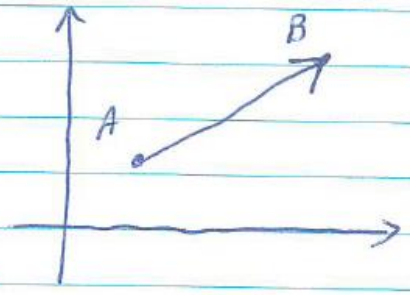
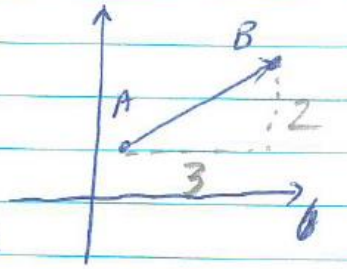
VECTORS.

Vector - A quantity which has a size and a direction.

Notation - The Vector from A to B. is written $\overrightarrow{AB}$
Sometimes vectors are represented by a single letter such as $\underset{\sim}{v}$.

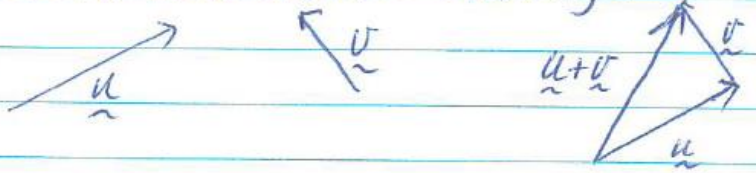


Column Vector - Represented as a column.
The column $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ corresponds to the vector which goes 3 units across and 2 units up.



Magnitude of Vector = It's length.
Written as $|\overrightarrow{AB}|$
In general if $\overrightarrow{AB}$ is the column vector $\begin{bmatrix} x \\ y \end{bmatrix}$
then $|\overrightarrow{AB}| = \sqrt{x^2+y^2}$

Addition of Vectors - Geometrically vectors are placed head to tail when adding.



For column vectors

if $\underset{\sim}{u} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

then $\underset{\sim}{u} + \underset{\sim}{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

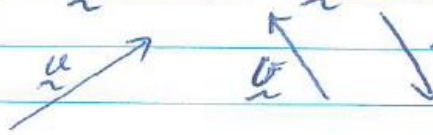
Scalar Multiplication – Changes the length of the vector.

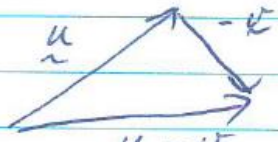
$2\underset{\sim}{u}$ is twice the length of $\underset{\sim}{u}$

When a vector is multiplied by a -ve the direction is reversed.

Subtraction of Vectors – Written in terms of addition then add geometrically.

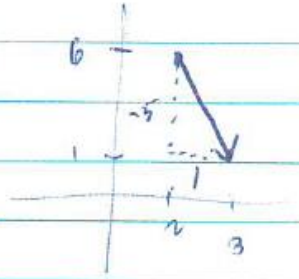
$\underset{\sim}{u} - \underset{\sim}{v} \rightarrow \underset{\sim}{u} + (-\underset{\sim}{v})$





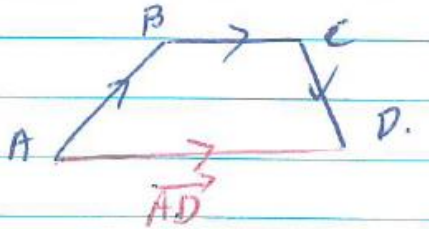
Eg The vector $\underset{\sim}{u}$ is defined as the line joining (2,6) to (3,1).

If $\underset{\sim}{u} = \begin{bmatrix} a \\ b \end{bmatrix}$ find a and b.



$u = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$

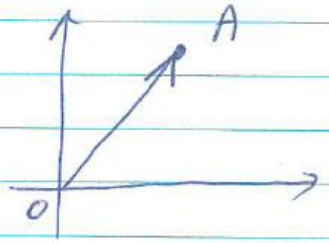
Eg Illustrate the vector sum $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$.



Parallel Vectors – Vectors are parallel if they point in the same or opposite direction
$\underset{\sim}{u}$ and $\underset{\sim}{v}$ parallel if $\underset{\sim}{u} = k\underset{\sim}{v}$.

Eg If $\underset{\sim}{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} -6 \\ -9 \end{bmatrix} = -3\begin{bmatrix} 2 \\ 3 \end{bmatrix}$
then $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are parallel.

Position Vectors – Locate a point in space relative to the origin.



For point A, the position vector is $\overrightarrow{OA}$

Vectors in 3-Dimensions –
Represented in Column Form as $\underset{\sim}{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$

Linear Dependence.

Two vectors are linearly dependent if they are parallel, $\underset{\sim}{a} = k\underset{\sim}{b}$, otherwise they are linearly independent. $k \neq 0$.

Any three non parallel vectors are linearly dependent if

$\underset{\sim}{c} = m\underset{\sim}{a} + n\underset{\sim}{b}$, $m + n$ are not both zero.